**A Comparative Analysis of Waiting Lines in the Nigerian Banking Industry**

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## Abstract

Waiting for services is part of our daily routine and it is a common experience in virtually every economic life. There is hardly any economic activity that waiting line is not essential. The major objective of the study is to compare the outcome of the application of queuing model/theory to Automated Teller Machine services of First bank of Nigeria, Access bank and United bank for Africa at Coker branch, Lagos, Nigeria. The specific objectives are to determine the mean number of arrivals per hour ($\lambda$) at the Automated teller machine facility of the three banks; examine the mean number of their customers served per hour ($\mu$); analyse the relationship between the mean number of arrivals and the mean number of customers served per hour ($\lambda$ and $\mu$) in the three banks and evaluate the average time a customer spends waiting in the queue before being served by a facility. The study population comprised of 349, 530 and 431 customers, for First Bank, Access Bank and UBA Bank respectively and the basic data were collected using observational timing of customer entry and service over a period of two weeks. The chi-square goodness of fit test was used to test the arrival pattern to determine if it follows a Poisson distribution and also tested the service pattern to determine if it follows an exponential distribution. The results obtained from the chi-square test showed that the arrival pattern follows a Poisson distribution and that the service pattern follows an exponential distribution, hence it can be analyzed using Markovian process. The raw data were then analyzed using Excel template bearing the multichannel and two servers queue model equations. Findings revealed that Access bank have the highest arrival of customers with an arrival rate of 40 customers per hour compared to First bank and UBA with 36 and 32 customers per hour respectively. The service rate of the three banks for the first facility is 32, 34, and 33 customers per hour respectively, and the second facility is 34, 33 and 35 customers per hour respectively. This shows that for the first facility, Access bank serve more customers efficiently than first bank and UBA, and for the second facility, UBA serve more customers than First bank and Access bank. UBA with utilization factor of 47% of time can be considered efficient than First bank and Access bank with utilization factor of 54.2% and 59.5% of time respectively. Thus, it can be deduced from this analysis that queuing theory is a good measure of efficiency.

**Keywords**: Queue, Arrival pattern, Service pattern, Customers, Servers

# 1 Introduction

## 1.1 Background Information

Waiting for services is part of our daily routine and it is a common experience in virtually every economic life. There is hardly any economic activity that waiting line is not essential. Everyone has experienced waiting in line, whether at a fast food restaurant, on the phone for technical help, at the doctor's office or in the drive through lane of a bank. Customers wait on line to get attention of the Cashiers in the Banks and attendants at the filing stations, bus stops, supermarkets, traffic light, telephone booth, and Automated teller machine facility, also student in registration line. Queues are necessary evil in any organized society. The more society becomes interdependent psychologically, economically and technically, the more individuals encounter waiting lines, or queues in their daily lives. They are formed primarily because the arrival time of someone who needs a service and the time of someone or facility to provide the service vary from a predetermined schedule. Since a queue forms whenever current demand exceeds the existing capacity to serve when each counter is so busy, thus arriving customers cannot receive immediate service facility. So each server process is done as a queuing model in this situation.

Queuing model is an application of a quantitative model to customers flow management. The theory enables mathematical analysis of several related processes, including arriving at the back of the queue, waiting in the queue and being served by the service facility server(s) at the front of the queue (Taha, 2007). Waiting is a non-value added activity. No customer likes a waiting situation.(Siddahartan, K; Jones , W J; Johnson, J A, 1996) described a queuing system by its input or arrival process, its queue discipline, and its service mechanism. Therefore, it is always a desire of every customer to obtain an efficient and prompt service delivery from a service system. There is therefore the need for effective and efficient management of queues for optimum service delivery in Organizations. Given the intensity of competition today, a customer waiting too long in line is potentially a lost customer. An efficient bank pays much attention to arrivals, service times and the order in which arriving customers are served in order to boost patronage. Understanding the nature of lines or queues and learn how to manage them is one of the most important areas in operations management. Queuing systems are described by distribution of service times, the number of servers, the service discipline, and distribution of inter- arrival times and the maximum capacity e.t.c.

Automated Teller Machine indicates the development of information technology in banking sector. It is a computerized telecom device designed to provide effective and efficient financial services to bank customers at the shortest possible time. The service system is sometimes hampered by rowdiness of its customers and their random arrival and service time. This scenario in banks makes customers to form a queue system for an orderly service performance. The major problem faced by these Automated Teller Machines are the long queue of customers at the peak hours and then at the off peak hours the lack of customer entry. The number of customer at most times might be large that customers wait for more than half an hour to get his or her turn but the facilities remain idle that there are no customers to serve depending on the current capacity of each Automated teller machine; many alternate decisions can be made. Thus the problem of Automated teller machine facility is significant.

## 1.2 Research Objectives

This study aim at apply queuing model/theory to Automated teller machine services of First bank of Nigeria, Access bank and United bank for Africa at Coker branch, Lagos, Nigeria.

The specific objectives are to:

i. determine the mean number of arrivals per hour (λ) at the Automated teller machine facility of the three banks.
ii. examine the mean number of their customers served per hour (μ).
iii. analyse the relationship between the mean number of arrivals and the mean number of customers served per hour (λ and μ) in the three banks.
iv. evaluate the average time a customer spends waiting in the queue before being served by a facility.

## 2.0 Literature review

Queuing theory is the construction of mathematical models of the various types of queuing systems to predict how the system will cope with demand made upon it. Queues are formed when the rate of items requiring service are greater than the rate of service. The queue permits the derivation and calculation of several performance measures including the average waiting time in the queue or the system, the expected number waiting or receiving service and the probability of encountering the system in certain states such as empty, full, having an available server or having to wait a certain time to be served. Queuing theory has become one of the most important, valuable and arguable one of the most universally used tool by an operational researcher. It has applications in diverse fields including telecommunications, traffic engineering, computing and design of factories, shops, offices, banks and hospitals. A queuing model of a system is an abstract representation whose purpose is to isolate those factors that relate to the system's ability to meet service demands whose occurrences and durations are random. (Sztrik, 2000). The study of queue deals with quantifying the phenomenon of waiting in lines using representative measures of performance, such as average queue length, average waiting time in queue and average facility utilization (Taha, 2007). Singh, (2011) Some of the analysis that can be derived using queuing theory include the expected waiting time in the queue, the average waiting time in the system, the expected queue length, the expected number of customers served at one time, the probability of balking customers, as well as the probability of the system to be in certain states, such as empty or full. Houda, Taoufik, & Hichem, (2008) emphasized that waiting lines and service systems are important parts of the business world. In their article they described several common queuing situations and presented mathematical models for analyzing waiting lines following certain assumptions. Those assumptions are that arrivals come from an infinite or very large population, arrivals are Poisson distributed, arrivals are treated on a FIFO basis and do not balk or renege, service times follow the negative exponential distribution or are constant, and the average service rate is faster than the average arrival rate. According to Ford (1980) "Waiting lines develop when "clients" arriving for "service" are delayed prior to being served". If customers are scheduled to visit service facilities, and the scheduling rule strictly adhered to, queues can be avoided. Nevertheless, in reality this is not the case as most of the time customers arrive at these service facilities in a random and uncontrolled manner. Arriving customers who meet a busy server and/or a waiting line of customers, either departs or waits in the queue for his or her turn during which time the customer holds on to the server. After service is completed, customers are generally assumed to leave the system making it available for other customers. Unmanaged queues are detrimental to the gainful operation of service systems and results in a lot of other managerial problems. For instance an ATM that receives and accommodates huge inflow of customers can be detrimental to its smooth running and response time. A slow response would greatly affect the speed at which service is provided to customers. As a results service providers may loss customers who grow impatient and leave the system. Queuing models are used to represent the various types of queuing systems that arise in practice, the models enable in finding an appropriate balance between the cost of service and the amount of waiting (Nafees, 2007). Queuing models provide the analyst with a powerful tool for designing and evaluating the performance of queuing systems (Banks, Carson, Nelson, Nicol, 2001). Any system in which arrivals place demands upon a finite capacity resource maybe termed as queuing systems, if the arrival times of these demands are unpredictable, or if the size of these demands is unpredictable, then conflicts for the use of the resource will arise and queues of waiting customers will form.

Davis, (2003) assert that providing ever-faster service, with the ultimate goal of having zero customer waiting time, has recently received managerial attention for several reasons. First, in the more highly developed countries, where standards of living are high, time becomes more valuable as a commodity and consequently, customers are less willing to wait for service. Second, this is a growing realization by organizations that the way they treat their customers today significantly impact on whether or not they will remain loyal customers tomorrow. Finally, advances in technology such as computers, internet etc., have provided firms with the ability to provide faster services. Researchers have argued that service waits can be controlled by two techniques: operations management or perceptions management (Hall, 2006). The operation management feature deals with the organization of how customers (customers), queues and servers can be coordinated towards the goal of rendering efficient service at the minimum cost. The (Abdul-Wahab & Ussiph, 2014) act of waiting has significant impact on customers' satisfaction. The amount of time customers must spend waiting can significantly influence their satisfaction. Additionally, research has demonstrated that customer satisfaction is affected not just by waiting time but also by customer expectations or attribution of the causes for the waiting. Consequently, one of the issues in queue management is not only the actual amount of time the customer has to wait, but also the customer's perceptions of that wait. Clearly, there are two approaches to increasing customer satisfaction with regard to waiting time: through decreasing actual waiting time, as well as through enhancing customer's waiting experience (Singh, 2011).

In 1909, the first study of queuing theory was done by a Danish mathematician, A.K. Erlang which resulted into the worldwide acclaimed Erlang telephone model. He examined the Telephone network system and tried to determine the effect of fluctuating service demands on calls on utilization of automatic dial equipment. The original problem Erlang treated was the calculation of this delay for one telephone operator and in 1917; the results were extended to the activities of several telephone operators. That was the same year that Erlang published his well-known work "solution of some problem in the theory of probabilities of significance in Automatic Telephone exchanges." The Automated Teller Machine (ATM) is one of the several electronic banking channels used in the banking industry of Ghana. According to (Aldajani & Alfares, 2009), automated teller machines are among the most important service facilities in the banking industry. The development of Automated teller machine has gone through many stages, it started from its baby stage in the late 1930s and then geared up for longer runs in the 1960s, and finally a matured and stable stage that we see today. Undoubtedly, most of the ideas and patents contributed for makeover of the Automated teller machine from time to time form the backbone of what was initiated as "holes in the wall".

Today, Automated teller machines hold a strong foothold in the world, offering everyone a better access to their money, be it in any corner of the world. There are about 1.8 million Automated teller machines in use around the world with Automated teller machines on cruise and navy ships, airports, newsagents and petrol stations. Automated teller machines too have been categorized as on and off premise Automated teller machines. On premise Automated teller machines are capable to connect the users to the bank with multi-function capabilities. off premise, Automated teller machine machines on the other hand are the "white label automated teller machines" and are limited to cash dispense

## 2.1 Queue characteristics

Queuing systems are characterized by the following components:-

i. the arrival pattern of customers
ii. the service pattern of customers
i. the number of servers.
ii. the capacity of the facility to hold customers
iii. the order in which customers are served

## 2.2 Assumptions of the Model

The single-channel, single-phase model considered here is one of the most widely used and simplest queuing models. It involves assuming that seven conditions exits:

i. Arrivals are served on a FIFO basis.

ii. Every arrival waits to be served regardless of the length of the line; that is, there is no balking or reneging.

iii. Arrivals are independent of preceding arrivals, but the average number of arrivals (the arrival rate) does not change over time.

iv. Arrivals are described by a Poisson probability distribution and come from an infinite or very large population.

v. Service time also varies from one passenger to the next and are independent of one another, but their average rate is known.

vi. Both the number of items in queue at anytime and the waiting line experienced by a particular item are random variables.

vii. Service times occur according to the negative exponential probability distribution.

viii. The average service rate is greater than the average arrival rate.

ix. The waiting space available for customers in the queue is infinite

## 2.3 Limitations of queuing theory

i. No simultaneous arrivals are allowed.
ii. Limited to dealing with situation of finite customers.
iii. Arrival rate may vary with time, weather condition, occurrence of some unusual events. e.t.c
iv. Some customers may be turned away once the queue has reached a certain size.

# 3 Methodology

## 3.1 Study Area

This research was centred on the waiting area of the Automated teller machine of First bank, Access bank, and United bank for Africa located at Coker area of Lagos.

The reason for the choice of this bank is because of its busy schedule and the familiarity of the researcher with these banks.

## 3.2 Research design

The objectives of this research were achieved by a descriptive survey and analysis of the real life observed data, then constructing a new model of system and using statistical analytical tools like Poisson, exponential and chi-square distribution to study pattern and reaction to change in the system. The data was collected primarily by direct observation at the Automated teller machine queue of the three banks. Thus, the researcher recorded the following events as it happened in the system using a wrist watch.

1. The time of arrival of each customer.
2. The time service commences for each customer in the system.
3. The time the customer leaves the system.
4. The number of customers on the waiting line

These events were observed at the withdrawal section of the banking hall during the peak and off peak hours of the banks operation. A form was designed for this exercise and the above required information was recorded in the form. Three weeks of five working days was spent to collect relevant data making ten days in all. An infinite population was considered due to the nature of operation of these banks.

## 3.3 Data Analysis techniques

Chi-square distribution was used to study the pattern and reactions to change of the system i.e. identification of customers, server and queue characteristics that are apparent in the system. Chi-square is a unique statistical test designed to investigate the agreement of a set of observed frequency and expected frequency on the assumption of a theoretical model for the phenomenon being studied. The test was used for investigating dependency or independency of two attributes of classification.

A measure for the discrepancy existing between observed and expected frequency is supplied by the chi-square ($X^2$) statistic given by;

$$X^2 = \frac{\sum(f_n - fe_n)^2}{fe_n}$$

where $f_n$ = actual or observed frequency

$fe_n$ = expected frequency

After the data collected from the banks was analysed based on;

i. The time arrival of each customer.

ii. The time service commence for each customer and when each customer leaves the system using the M│M│S model since the system has to do with multiple servers and a single queue.

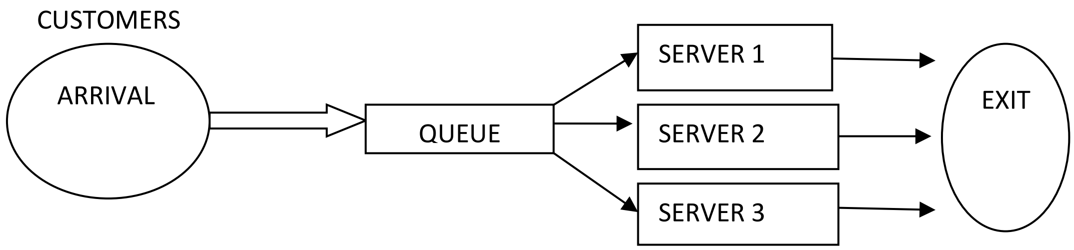


***Figure 3.1: Single stage Queuing Model with single Queue and Multiple parallel servers***

**Source:** *Sheu and Babbar (1996)*

## 4. Results and Discussion

Tables 4.1, 4.2 and 4.3 are the tables showing the number of arrivals of customers for each respective minute in the three banks under study

We now use the chi square test goodness of fit to test the hypothesis that

$H_O$ = Arrival distribution does not followed Poisson distribution

$H_1$ = Arrival distribution followed Poisson distribution

Let n = number of arrivals for each minute

$F_n$ = Observed frequency of number of arrivals

$fe_n$ = Expected frequency of number of arrivals

$P_n$ = Probability of number of arrivals

Table 4.1 FIRST BANK

| N | $F_n$ | $NF_n$ | $P_n$ | $fe_n$ | $f_n - fe_n$ | $(f_n - fe_n)^2$ | $\frac{(f_n - fe_n)^2}{e_n}$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0.16669 | 63.1755 | -63.1755 | 3991.4 | 63.1754 |
| 1 | 206 | 206 | 0.29863 | 113.1808 | 92.8192 | 8615.40 | 76.1207 |
| 2 | 94 | 188 | 0.26751 | 101.3863 | -7.3863 | 54.5574 | 0.5381 |
| 3 | 42 | 126 | 0.15975 | 60.5453 | -18.5453 | 343.9282 | 5.6805 |
| 4 | 26 | 104 | 0.01343 | 5.08997 | 20.91003 | 437.2294 | 85.9001 |
| 5 | 11 | 55 | 0.02564 | 9.7176 | 1.2824 | 1.6445 | 0.1692 |
| | **379** | **679** | | | | | **231.584** |

Source: Researchers Observation (2019)

$P_n = \frac{e^{-\lambda} - \lambda^n}{n!}$ Where $\lambda = \frac{\sum nf_n}{\sum f_n} = \frac{679}{379} = 1.79156$

$$P_n = \frac{e^{-1.79156} - 1.79156^n}{n!}$$

$fe_n = (\sum f_n)P_n = 379P_n$

$X^2 \text{ Cal} = \underline{\sum(f_n - fe_n)^2} = 231.584$

$fe_n$

To find the $X^2$ we check the distribution table.

Degree of freedom = number of observed value – number of estimated parameter – 1 = n – 1 – 1

Number of observed is 5 and the number of estimated parameter is 1

Hence the degree of freedom = 5-1-1= 3

Taking $\alpha = 5\%$, from the $X^2$ distribution table, 0.05 critical value of $X^2$ for 5 degree of freedom

$X^2$ tab = $X^2_{3,\, 0.05} = 7.815$

$X^2$ cal > $X^2$ tab. Hence $H_1$ is accepted and thus there is no reason on the basis of this test for doubting that queuing model can be applied to this data. This also implies that the arrival pattern follows a Poisson distribution.

TABLE 4.2 ACCESS BANK

| N | $F_n$ | $NF_n$ | $P_n$ | $fe_n$ | $f_n - fe_n$ | $(f_n - fe_n)^2$ | $\frac{(f_n - fe_n)^2}{e_n}$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0.1723 | 91.319 | -91.319 | 8339.16 | 91.319 |
| 1 | 241 | 241 | 0.3029 | 160.537 | 80.463 | 6474.29 | 40.329 |
| 2 | 150 | 300 | 0.2664 | 141.192 | 8.808 | 77.58 | 0.5495 |
| 3 | 76 | 111 | 0.1562 | 82.786 | -6.786 | 46.05 | 0.5563 |
| 4 | 37 | 148 | 0.0687 | 36.411 | 0.589 | 0.3469 | 0.0095 |
| 5 | 24 | 120 | 0.0241 | 12.774 | 11.226 | 126.02 | 9.8654 |
| 6 | 2 | 12 | 0.0071 | 3.763 | -1.763 | 3.108 | 0.8259 |
| | **530** | **932** | | | | | **143.4546** |

Source: Researchers Observation (2019)

$$P_n = \frac{e^{-\lambda} - \lambda^n}{n!} \qquad \text{Where } \lambda = \frac{\sum nf_n}{\sum f_n} = \frac{932}{530} = 1.7585$$

$$P_n = \frac{e^{-1.7585} - 1.7585^n}{n!}$$

$fe_n = (\sum f_n)P_n = 530P_n$

$$X^2 \text{ Cal} = \frac{\sum(f_n - fe_n)^2}{fe_n} = 143.4546$$

To find the $X^2$ we check the distribution table.

Degree of freedom = number of observed value – number of estimated parameter – 1 = n – 1 – 1

Number of observed is 6 and the number of estimated parameter is 1

Hence the degree of freedom = 6-1-1= 4

Taking α = 5%, from the $X^2$ distribution table, 0.05 critical value of $X^2$ for 5 degree of freedom

$X^2$ tab = $X^2_{4,\ 0.05}$ = 9.488

$X^2$ cal > $X^2$ tab. Hence $H_1$ is accepted and thus there is no reason on the basis of this test for doubting that queuing model can be applied to this data. This also implies that the arrival pattern follows a poisson distribution.

TABLE 4.3 UBA

| N | $F_n$ | $NF_n$ | $P_n$ | $fe_n$ | $f_n - fe_n$ | $(f_n - fe_n)^2$ | $\frac{(f_n - fe_n)^2}{e_n}$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0.1338 | 57.6678 | -57.6678 | 3325.58 | 57.6678 |
| 1 | 194 | 194 | 0.2691 | 115.9821 | 78.0179 | 6086.79 | 52.4804 |
| 2 | 117 | 234 | 0.2707 | 116.6717 | 0.3283 | 0.1078 | 0.00092 |
| 3 | 62 | 186 | 0.1815 | 78.2265 | -16.2265 | 263.299 | 3.3659 |
| 4 | 37 | 148 | 0.0913 | 39.3503 | -2.3503 | 5.5239 | 0.1404 |
| 5 | 21 | 105 | 0.0367 | 15.8177 | 5.1823 | 26.8562 | 1.6983 |
| | **431** | **867** | | | | | **115.3537** |

Source: Researchers Observation (2019)

$$P_n = \frac{e^{-\lambda} - \lambda^n}{n!} \qquad \text{Where } \lambda = \frac{\sum nf_n}{\sum f_n} = \frac{867}{431} = 2.0116$$

$$P_n = \frac{e^{-2.0116} - 2.0116^n}{n!}$$

$fe_n = (\sum f_n)P_n = 431P_n$

$$X^2 \text{ Cal} = \frac{\sum(f_n - fe_n)^2}{fe_n} = 115.3537$$

To find the $X^2$ we check the distribution table.

Degree of freedom = number of observed value – number of estimated parameter – 1 = n – 1 – 1

Number of observed is 5 and the number of estimated parameter is 1

Hence the degree of freedom = 5-1-1= 4

Taking α = 5%, from the $X^2$ distribution table, 0.05 critical value of $X^2$ for 5 degree of freedom

$X^2$ tab = $X^2_{3,\ 0.05}$ = 7.815

$X^2$ cal > $X^2$ tab. Hence $H_1$ is accepted and thus there is no reason on the basis of this test for doubting that queuing model can be applied to this data. This also implies that the arrival pattern follows a Poisson distribution.

We now use the chi square to test goodness of fit that as presented in Tables 4.4, 4.5 and 4.6

$H_O$ = Service time distribution is not exponential

$H_1$ = Service time distribution is exponential

Let T = service time in minute

$F_n$ = Observed frequency of the service times

$fe_n$ = Expected frequency of the service times

$P_n$ = Probability of service time

Table 4.4

SERVER 1 (FIRST BANK)

| T | $F_n$ | $P_n$ | $fe_n$ | $f_n - fe_n$ | $(f_n - fe_n)^2$ | $\frac{(f_n - fe_n)^2}{fe_n}$ |
|---|---|---|---|---|---|---|
| $0 \leq T < 2$ | 320 | 0.6965 | 405.363 | -85.363 | 7286.842 | 17.9761 |
| $2 \leq T < 4$ | 184 | 0.2114 | 123.035 | 60.965 | 3716.731 | 30.2087 |
| $4 \leq T < 6$ | 78 | 0.0642 | 37.364 | 40.636 | 1651.284 | 44.1945 |
| | **582** | | | | | **92.3793** |

$fe_n = \sum f_n(p_n) = 582P_n$

$P_n = \mu.e^{u\text{-}T}$

Where $\mu = \dfrac{\text{system capacity}}{\text{Time taken to be served}}$

$\mu = \frac{582}{760} = 0.7658$

$P(a \leq T < b) = \int_a^b \mu.\, e^{\mu\text{-}T} dT$

$-e^{-\mu b} + e^{-\mu a}$

$e^{-\mu a} - e^{-\mu b}$

$P(0 \leq T<2) = 2.1783^{-0.7658(0)} - 2.1783^{-0.7658(2)} = 0.6965$

$P(2 \leq T<4) = 2.1783^{-0.7658(2)} - 2.1783^{-0.7656(4)} = 0.2114$

$P(4 \leq T<6) = 2.1783^{-0.7658(4)} - 2.1783^{-0.7658(6)} = 0.0642$

$X^2$ Cal = $\sum \frac{(f_n - fe_n)^2}{fe_n}$ = 92.3793

to find $X^2$tab, we check the chi square distribution table

degree of freedom = 3-1-1 = 1

taking α = 5%, from the $X^2$ distribution table, 5% critical value of $X^2$ for 1 degree of freedom is

$X^2$tab = $X^2_{1,\, 0.05}$ = 3.841

$X^2$ cal > $X^2$ tab. Therefore $H_1$ is accepted, and this connote that the service time pattern for Server 1 follows an exponential distribution.

TABLE 4.5 SERVER 2(FIRST BANK)

| T | $F_n$ | $P_n$ | $fe_n$ | $f_n - fe_n$ | $(f_n - fe_n)^2$ | $\frac{(f_n - fe_n)^2}{e_n}$ |
|---|---|---|---|---|---|---|
| $0 \leq T < 2$ | 326 | 0.6965 | 405.363 | -79.363 | 6298.486 | 15.5379 |
| $2 \leq T < 4$ | 172 | 0.2114 | 123.035 | 48.965 | 2397.571 | 19.4869 |
| $4 \leq T < 6$ | 84 | 0.0642 | 37.364 | 46.636 | 2174.916 | 58.2089 |
| | **582** | | | | | **77.6958** |

$X^2$ Cal = $\sum \frac{(f_n - fe_n)^2}{fe_n}$ = 77.6958

to find $X^2$tab, we check the chi square distribution table

degree of freedom = 3-1-1 = 1

taking α = 5%, from the $X^2$ distribution table, 5% critical value of $X^2$ for 1 degree of freedom is

$X^2$tab = $X^2_{1,\, 0.05}$ = 3.841

$X^2$ cal > $X^2$ tab. Therefore $H_1$ is accepted, and this connote that the service time pattern for Server 2 follows an exponential distribution.

TABLE 4.6 SERVER 1 (ACCESS BANK)

| T | $F_n$ | $P_n$ | $fe_n$ | $f_n - fe_n$ | $(f_n - fe_n)^2$ | $\frac{(f_n - fe_n)^2}{fe_n}$ |
|---|---|---|---|---|---|---|
| $0 \leq T < 3$ | 312 | 0.8483 | 565.816 | -253.816 | 64422.56 | 113.858 |
| $3 \leq T < 6$ | 254 | 0.1287 | 85.843 | 168.157 | 28276.78 | 329.401 |
| $6 \leq T < 9$ | 101 | 0.0195 | 13.007 | 87.993 | 7742.77 | 595.277 |
| | **667** | | | | | **1038.536** |

$fe_{n} = \sum f_n(p_n) = 667P_n$

$P_n = \mu.e^{u\text{-}T}$

Where $\mu = \dfrac{\text{system capacity}}{\text{Time taken to be served}}$

$\mu = \frac{667}{826} = 0.8075$

$P(a \leq T < b) = \int_a^b \mu.\, e^{\mu\text{-}T} dT$

$-e^{-\mu b} + e^{-\mu a}$

$e^{-\mu a} - e^{-\mu b}$

$P(0 \leq T<3) = 2.1783^{-0.8075(0)} - 2.1783^{-0.8075(3)} = 0.8483$

$P(3\leq T<6) = 2.1783^{-0.8075(2)} - 2.1783^{-0.8075(4)} = 0.1287$

$P(4\leq T<6) = 2.1783^{-0.8075(6)} - 2.1783^{-0.8075(9)} = 0.0195$

$X^2 \text{ Cal} = \dfrac{\sum(f_n - fe_n)^2}{fe_n} = 1038.536$

to find $X^2$tab, we check the chi square distribution table

degree of freedom = 3-1-1 = 1

taking α = 5%, from the $X^2$ distribution table, 5% critical value of $X^2$ for 1 degree of freedom is

$X^2\text{tab} = X^2_{1,\,0.05} = 3.841$

$X^2$ cal > $X^2$ tab. Therefore $H_1$ is accepted, and this connote that the service time pattern for Server 1 follows an exponential distribution.

TABLE 4.7 SERVER 2 (ACCESS BANK)

| T | $F_n$ | $P_n$ | $fe_n$ | $f_n - fe_n$ | $(f_n - fe_n)^2$ | $\frac{(f_n - fe_n)^2}{e_n}$ |
|---|---|---|---|---|---|---|
| $0 \leq T < 3$ | 292 | 0.8483 | 565.8161 | -273.8161 | 74975.26 | 132.5081 |
| $3 \leq T < 6$ | 224 | 0.1287 | 85.8429 | 138.1571 | 19087.38 | 222.3525 |
| $6 \leq T < 9$ | 151 | 0.0195 | 13.0065 | 137.9935 | 19042.21 | 1464.0534 |
| | **667** | | | | | **1818.914** |

$X^2 \text{ Cal} = \dfrac{\sum(f_n - fe_n)^2}{fe_n} = 1818.914$

to find $X^2$tab, we check the chi square distribution table

degree of freedom = 3-1-1 = 1

taking α = 5%, from the $X^2$ distribution table, 5% critical value of $X^2$ for 1 degree of freedom is

$X^2tab = X^2_{1, 0.05} = 3.841$

$X^2$ cal > $X^2$ tab. Therefore $H_1$ is accepted, and this connote that the service time pattern for Server 2 follows an exponential distribution.

TABLE 4.8 SERVER 1 (UBA)

| T | $F_n$ | $P_n$ | $fe_n$ | $f_n - fe_n$ | $(f_n - fe_n)^2$ | $\frac{(f_n - fe_n)^2}{fe_n}$ |
|---|---|---|---|---|---|---|
| $0 \leq T < 2$ | 342 | 0.7213 | 481.8284 | -139.8284 | 19551.98 | 40.5787 |
| $2 \leq T < 4$ | 201 | 0.20101 | 134.2747 | 66.7253 | 4452.27 | 33.1579 |
| $4 \leq T < 6$ | 125 | 0.04069 | 27.1809 | 97.8191 | 9568.58 | 352.0332 |
| | **668** | | | | | **425.7698** |

$fe_n = \sum f_n(p_n) = 668P_n$

$P_n = \mu . e^{u-T}$

Where $\mu = \frac{\text{system capacity}}{\text{Time taken to be served}}$

$\mu = \frac{668}{814} = 0.8206$

$P(a \leq T < b) = \int_a^b \mu . e^{\mu-T} dT$

$-e^{-\mu b} + e^{-\mu a}$

$e^{-\mu a} - e^{-\mu b}$

$P(0 \leq T<2) = 2.1783^{-0.8206(0)} - 2.1783^{-0.8206(2)} = 0.72134$

$P(2 \leq T<4) = 2.1783^{-0.8206(2)} - 2.1783^{-0.8206(4)} = 0.20101$

$P(4 \leq T<6) = 2.1783^{-0.8206(4)} - 2.1783^{-0.8206(6)} = 0.04069$

$X^2 \text{ Cal} = \frac{\sum(f_n - fe_n)^2}{fe_n} = 425.7698$

to find $X^2$tab, we check the chi square distribution table

degree of freedom = 3-1-1 = 1

taking α = 5%, from the $X^2$ distribution table, 5% critical value of $X^2$ for 1 degree of freedom is

$X^2tab = X^2_{1, 0.05} = 3.841$

$X^2$ cal > $X^2$ tab. Therefore $H_1$ is accepted, and this connote that the service time pattern for Server 2 follows an exponential distribution.

TABLE 4.9 SERVER 2 (UBA)

| T | $F_n$ | $P_n$ | $fe_n$ | $f_n - fe_n$ | $(f_n - fe_n)^2$ | $\frac{(f_n - fe_n)^2}{e_n}$ |
|---|---|---|---|---|---|---|
| $0 \leq T < 2$ | 302 | 0.7213 | 481.8284 | -179.8284 | 32338.25 | 67.116 |
| $2 \leq T < 4$ | 264 | 0.20101 | 134.2747 | 129.7253 | 16828.65 | 125.330 |
| $4 \leq T < 6$ | 102 | 0.04069 | 27.1809 | 74.8191 | 5597.897 | 205.949 |
| | **668** | | | | | **398.395** |

$X^2$ Cal $= \sum \frac{(f_n - fe_n)^2}{fe_n} = 398.395$

to find $X^2$tab, we check the chi square distribution table

degree of freedom = 3-1-1 = 1

taking $\alpha = 5\%$, from the $X^2$ distribution table, 5% critical value of $X^2$ for 1 degree of freedom is

$X^2$tab $= X^2_{1, 0.05} = 3.841$

$X^2$ cal > $X^2$ tab. Therefore $H_1$ is accepted, and this connote that the service time pattern for Server 2 follows an exponential distribution.

**Estimation of parameters**

From the data collected, the total number of customers sampled (N) were 500 customers.

FIRST BANK

Inter arrival time for 500 customers is 835 minutes.

Time taken for 500 customers to be served by the first facility (SERVER 1) is 928 minutes.

Time taken for 500 customers to be served by the second facility (SERVER 2) is 885 minutes.

Thus,

Arrival rate ($\lambda$) = $\frac{500}{835}$ = 0.5988 customer per minute i.e 36 customers per hour

Service rate$_1$ ($\mu_1$) = $\frac{500}{928}$ = 0.5388 customer per minute i.e 32 customers per hour

Service rate$_2$ ($\mu_2$) = $\frac{500}{885}$ = 0.56497 customer per minute i.e 34 customers per hour

This implies that,

$\lambda = 0.6609$

$\mu_1 = 0.5388$

$\mu_2 = 0.56497$

S = 2

$$\mu = \frac{\mu_1 + \mu_2}{2} = \frac{0.5388 + 0.56497}{2} = 0.5519$$

## Model 3 (M/M/s Queue):

**Multiple servers, Infinite population, Poisson arrival, FCFS, Exponential service time, Unlimited waiting room**

### Inputs

| | | | |
|---|---|---|---|
| **Unit of time** | **Minute** | | |
| **Arrival rate (λ)** | 0.5998 | **customers per** | **minute** |
| **Service rate (μ)** | 0.5519 | **customers per** | **minute** |
| **Number of identical servers (s)** | 2 | **Servers** | |

### Outputs

| | | |
|---|---|---|
| **Mean time between arrivals** | 1.670 | |
| **Mean time per service** | 1.8119 | **Minute** |
| **Traffic intensity** | 0.5425 | **Minute** |

### Summary measures

| | | |
|---|---|---|
| **Average utilization rate of server (P)** | 54.2% | |
| **Average number of customers waiting in line ($L_q$)** | 0.4525 | **Customers** |
| **Average number of customers in system ($L_s$)** | 1.5374 | **Customers** |
| **Average time waiting in line ($T_q$)** | 0.75561 | **Minute** |
| **Average time in system ($T_s$)** | 2.56754 | **Minute** |
| **Probability of no customers in system (P0)** | 0.29661 | **(this is the probability of empty system)** |

From the above,

The probability that the servers are idle ($P_0$) = 0.29661, the expected average number in the waiting line ($L_q$) = 0.4525, the expected average number in the system {waiting plus in service} ($L_s$) = 1.5374, the expected average waiting time in the queue ($T_q$) =0.75561 minute.

ACCESS BANK

Inter arrival time for 500 customers is 756 minutes.

Time taken for 500 customers to be served by the first facility (SERVER 1) is 892 minutes.

Time taken for 500 customers to be served by the second facility (SERVER 2) is 906 minutes.

Thus,

Arrival rate ($\lambda$) = $\frac{500}{756}$ = 0.66137 customer per minute i.e 40 customers per hour

Service rate$_1$ ($\mu_1$) = $\frac{500}{892}$ = 0.5605 customer per minute i.e 34 customers per hour

Service rate$_2$ ($\mu_2$) = $\frac{500}{906}$ = 0.55187 customer per minute i.e 33 customers per hour

This implies that,

$\lambda = 0.66137$

$\mu_1 = 0.5605$

$\mu_2 = 0.55187$

S = 2

$$\mu = \frac{\mu_1 + \mu_2}{2} = \frac{0.5605 + 0.55187}{2} = 0.55619$$

## Model 3 (M/M/s Queue):

**Multiple servers, Infinite population, Poisson arrival, FCFS, Exponential service time, Unlimited waiting room**

### Inputs

| | | |
|---|---|---|
| **Unit of time** | **Minute** | |
| **Arrival rate ($\lambda$)** | 0.66137 | **customers per minute** |
| **Service rate ($\mu$)** | 0.55619 | **customers per minute** |
| **Number of identical servers (s)** | 2 | **Servers** |

### Outputs

| | | |
|---|---|---|
| **Mean time between arrivals** | 1.512 | |
| **Mean time per service** | 1.7979 | **Minute** |
| **Traffic intensity** | 0.5946 | **Minute** |

### Summary measures

| | | |
|---|---|---|
| **Average utilization rate of server (P)** | 59.5% | |
| **Average number of customers waiting in line ($L_q$)** | 0.65018 | **Customers** |
| **Average number of customers in system ($L_s$)** | 1.83929 | **Customers** |
| **Average time waiting in line ($T_q$)** | 0.98308 | **Minute** |
| **Average time in system ($T_s$)** | 2.78102 | **Minute** |
| **Probability of no customers in system ($P_0$)** | 0.25427 | |

From the above,

The probability that the servers are idle ($P_0$) = 0.25427, the expected average number in the waiting line ($L_q$) = 0.27579, the expected average number in the system {waiting plus in service} ($L_s$) = 1.22483, the expected average waiting time in the queue ($T_q$) = 0.51352.

UBA

Inter arrival time for 500 customers is 931 minutes.

Time taken for 500 customers to be served by the first facility (SERVER 1) is 913 minutes.

Time taken for 500 customers to be served by the second facility (SERVER 2) is 856 minutes.

Thus,

Arrival rate ($\lambda$) = $\frac{500}{931}$ = 0.53706 customer per minute i.e 32 customers per hour

Service rate$_1$ ($\mu_1$) = $\frac{500}{913}$ = 0.54764 customer per minute i.e 33 customers per hour

Service rate$_2$ ($\mu_2$) = $\frac{500}{906}$ = 0.55187 customer per minute i.e 35 customers per hour

This implies that,

$\lambda = 0.53706$

$\mu_1 = 0.54764$

$\mu_2 = 0.58411$

$S = 2$

$$\boldsymbol{\mu} = \frac{\mu_1 + \mu_2}{2} = \frac{0.54764 + 0.58411}{2} = 0.5659$$

The probability that the servers are idle ($P_0$) = 0.35638, the expected average number in the waiting line ($L_q$) =, the expected average number in the system {waiting plus in service} ($L_s$) = 1.83929, the expected average waiting time in the queue ($T_q$) = 0.98308.

**Table 4.10 COMPARATIVE ANALYSIS RESULT**

| BANKS | Arrival rate (customer per hour) | Service rate 1 (customer per hour) | Service rate 2 (customer per hour) | Utilization factor (%) |
|---|---|---|---|---|
| First bank | 36 | 32 | 34 | 54.2 |
| Access bank | 40 | 34 | 33 | 59.5 |
| UBA | 32 | 33 | 35 | 47 |

The table above shows the comparative result of these banks in terms of the arrival rate, service rate and utilization factor.

Access bank can be considered to have the highest arrival of customers with an arrival rate of 40 customers per hour than First bank and UBA with 36 customers per hour and 32 customers per hour respectively.

The service rate of the three banks for the first facility is 32, 34, and 33 customers per hour respectively, and the second facility is 34, 33 and 35 customers per hour respectively. This shows that for the first facility, Access bank serve more customers efficiently than first bank and UBA, and for the second facility, UBA serve more customers than First bank and Access bank.

Generally, UBA with utilization factor of 47% of time can be considered efficient than First bank and Access bank with utilization factor of 54.2% and 59.5% of time respectively.

Thus, it can be deduced from this analysis that queuing theory is a good measure of efficiency.

## 5 Conclusion and Recommendation

### 5.1 Conclusion

Queue theory is very useful and relevant for management of business organizations especially in the service industry. It is highly useful in planning and making decisions that will affect the smooth running of business organizations and enhance customer satisfaction. The involvement of human factor in the provision of service affects its variation. Henceforth, high human involvement in a system leads to high variation in the service duration and the lower the human involvement in system, the lower the service variation. One needs to recall that constant time is common when a service is highly mechanized or automated. The management of these Banks Plc must be conscious that if the queue formed is very large, it will discourage customers and this result to lose of customers to rival banks especially at this period that many banks have commenced operations. This therefore necessitates the bank to observe queue discipline, that is, customers must be served on first come and first served basis. Although, there are other queue disciplines such as Last In First Out, Alphabetical Discipline, etc. In some cases there are some pre-arranged schedules which is common among doctors. In this situation, customers will be attended to on predetermined arrangement not minding the time of arrival. However, management of the bank must as much as possible use FIFO method that is First Come First Out in attending to their customers. The operator of a service organization should know that queuing theory alone cannot bring solution to congestion problem in an organization. It is not an end but a means to an end. Therefore, manager cannot substitute it with managerial thinking; it can only serve as an aid to congestion problem. Henceforth, the management of the bank must effectively analyze the customers' situation and then complement it with queuing theory. Thus, this study is in support of the work of Odunukwe, (2013) which shows that queuing theory can be used to model bank settings as well as (Abdul-Wahab & Ussiph, 2014) and where it was also applied to Automated Teller Machine.

## 5.2 Recommendation

Since the utilization factor (PK) of the bank's Automated Teller Machine staging a single queue and multiple servers is greater than 0.5 for the facilities of both First bank and Access bank, it is recommended that the management of these banks should add one more server. This will help reduce the time customers spend on the queue and as well help to reduce the cost incurred from waiting. The bank (UBA) with utilization factor of it facilities less than 0.5 can also put into consideration other qualitative factors such as holidays that may affect the service quality in the long run.

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