

The Stochastic Forecasting of Stock Prices in Nigeria: Application of Geometric Brownian Motion (GBM) Model

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Abstract

In this study, the stochastic price movements of stocks are modeled by a geometric Brownian motion (GBM). The model assumptions of the GBM with drift: continuity, normality and Markov tendency, were investigated using four years (2015 - 2018) of historical closing prices of ten stocks listed on The Nigeria Stock Exchange. The sample for this study is based on the most continuously traded stocks. The forecasted daily closing prices were compared to actual prices in order to evaluate the accuracy of the prediction model. The results show that in the simulation there are some actual stock prices located outside trajectory realization that may be from GBM model. Thus, the model did not predict accurately the price behaviour of some of the listed stocks. The predictive power of the model is declining towards the longer the evaluated time frame proven by the higher value of the mean absolute percentage error. The value of the MAPE is 50% and below for the one- to two-year holding periods, and above 50% for the three-year holding period. The MAPE and directional prediction accuracy method provide support that over short periods the GBM model is accurate. Meaning that the GBM is a reasonable predictive model for one or two years, but for three years, therefore, it is an inaccurate predictor. It is recommended that the technical analyst whose primary motive is to make gain at the expense of other participants should identify high volatile portfolio in any holding period for effective prediction. Investors with long-range holding position as investment strategy should concentrate more on low capitalized stocks rather than stocks with large market capitalization.

Keywords: *Stochastic forecasting, Geometric Brownian motion, Stock return, The Nigerian stock Exchange*

Introduction

The stock market is the meeting place of both buyers and sellers (wider domain of trading activities) of stocks. It is a platform for investors to own shares of companies. The stock price is always fluctuating (constantly changing), and investors seek to know the future price of their investment and the risk associated with this investment. This motivation is understandable since they demand the certain return of their investment (Bohdalova and Gregus, 2012). Forecasting is the best method to know the future price of a stock (Omar and Jaffar, 2014). To forecast is to form an expectation of what will happen in the future. Two common approaches to predicting stock prices are those based on the theory of technical analysis and those based on the theory of fundamental analysis (Fama, 1995). Fundamental analysis assumes that the price of a stock depends on the intrinsic value and expected return on investment; while the technical analysis studies the price movement of a stock and predicts its future price movement (uses past stock prices and volumes); history will repeat itself. There are alternative approaches to the forecasting of stock prices, e.g., the Random Walk Theory. The Random Walk Theory is the idea that stocks prices take a random and unpredictable path, making it near impossible to outperform the market without assuming additional risk (Fama, 1970; 1991).

This random (zig-zag) movements of stock prices is referred to in finance as Brownian motion; and the Brownian motion model is used to capture the uncertainty in the returns of risky assets, (Bachelier, 1900; Maraddin and Trimono, 2018; Reddy and Clinton, 2016). Modeling stock price changes with Stochastic Differential

Equation (SDE) leads to Geometric Brownian Motion (GBM) model (Samuelson, 1965). The return on the risky asset, however, is uncertain; and this uncertainty or randomness is claimed to be captured by the Brownian motion models. Recent findings (mostly in developed countries) have shown that the GBM model is unable to capture some features including long range correlations and heavy-tailed distributions (Brigo, et al. 2007) and does not account for periods of constant values (Gadja & Wylomanska, 2012). We therefore sought to know if the GBM model as a stochastic Markov process, can be accurate when used to model and forecast future stock prices in The Nigeria stock Exchange.

The purpose of the study is to model and simulate the trends and behavioural patterns in The Nigerian Stock Market and hence predict the future stock prices within the Geometric Brownian Motion (GBM) framework. The specific objectives can be summarized as follows: (i) To investigate the extent in which the actual prices of individual stocks differ from those that are simulated using GBM model over the sample period; (ii) the relationship between actual and simulated prices for portfolios classified by volatility; (iii) the relationship between actual and simulated stock prices for portfolios classified by expected return; and (iv) the relationship between actual and simulated prices for portfolios classified by market capital. The hypotheses to be tested for the stock returns are the following: (1) the actual prices of the individual stocks are not significantly different from the simulated using GBM over the sample period; (2) there is no significant relationship between actual and predicted prices for portfolios classified by volatility; (3) there is no significant relationship between actual and predicted prices for portfolios classified by expected return; and (4) there is no significant relationship between actual and predicted prices for portfolios classified by market capital. The remainder of the paper is organized as follows: section 2 is the literature review, and section 3 describes the data and methodology. Section 4 shows the analysis and results while Section 5 offers some concluding remarks.

Overview of the Nigerian Capital Market

The Nigerian Capital Market is a channel of mobilizing long-term funds by providing mechanism for private and public savings through financial instruments (equities, debentures, bonds and stocks) with major components consisting of the Security and Exchange Commission (SEC) and the Nigerian Stock Exchange (NSE). Founded in 1960, the NSE is the second largest market in sub-Saharan Africa with fully automated exchange that provides the listing and trading services as well as electronic Clearing, Settlement and Delivery (CSD) services through Central Securities Clearing System (CSCS). The exchange keeps on evolving as a competitive market and meeting the needs of investors. It operates fair, orderly and transparent markets with over 200 listed equities and 258 listed securities, and had attracted the best of African enterprises as well as the local and global investors (NSE, 2013). The market has become an integral part of the global economy such that any shock in the market has contagious consequences. Moreover, the Nigeria's capital market has enjoyed a decade of unprecedented growth. The market capitalization increased by over 90.0% from 2003 to 2008. However, from a peak in March 2008, the market capitalization went declined spirally by about 46% in 2009 (SEC Report, 2009). The convergence of global economy makes all countries and all markets sensible to the happenings in other countries (the contagious effect). The 2008 global financial meltdown originated from the United States of America (USA) had varying degree of impacts on different capital markets in various countries. This situation is compounded with the continuous volatility in the global oil price which in theory adversely and significantly affecting capital markets (Njiforti, 2015; Asaolu and Ilo, 2016). Nigeria recently experienced economic recession as a consequence of the 2014-2016 global oil price downturn. In view of these, the various Security and Exchange Commission (SEC) reports came with several recommendations to reposition the Market as a world class institution. The main recommendations are; the development of an enforcement framework to prevent market manipulation, and the establishment of principles for risk management for capital market operators. The Nigerian Stock Exchange ("The Exchange" or "NSE") operates fully electronic marketplaces for Equities, Bonds,

Exchange Traded Products, with plans to include Derivatives trading shortly. The NSE operates an Automated Trading System (ATS) platform with a central order book which allows Dealing Members to participate on equal terms, competing on the hierarchical basis of Price, Cross and Time priority. The Exchange runs a hybrid market, allowing Dealing Members to submit orders and Market Makers to submit two-sided quotes into the order book (NSE, 2019).

Theoretical Framework -

Four main schools of thought have presented varied positions on how monetary policy actions explain the cyclical upswings and downswings in macroeconomics.

The Overview

A theory, according to Stoner, Freeman and Gilbert Jr. (2009), is a coherent group of assumptions put forth to explain the relationship between two or more observable facts and to provide a sound basis for predicting future events. More also, Kerlinger (1993) defined a theory as a set of interrelated constructs (concepts), definitions and propositions that present a systematic view of phenomena, by specifying relations among variables, with the purpose of explaining and predicting phenomena. Therefore, an understanding of the theories of geometric Brownian motion is strategic in helping us know where we are coming from and at the same time challenge us to keep learning about our field – quantitative finance.

Bachelier (1900) seems to be the first to have provided an analytical valuation for stock options. His work is rather remarkable because by addressing the problem of option pricing, Bachelier (1900) derived most of the theory of diffusion processes. The mathematical theory of Brownian motion has been formulated by Bachelier (1900) five years before Einstein's classic paper (Einstein 1905). Bachelier (1900) has formulated *avant la lettre* the Chapman-Kolmogorov equation (von Smoluchowski 1906; Chapman 1916; Chapman 1917; Kolmogorov 1931), called today the Chapman-Kolmogorov-Smoluchowski-Bachelier equation (Brown et al. 1995), and the Fokker-Plank or Kolmogorov equation (von Smoluchowski 1906; Fokker 1914; Fokker 1917; Plank 1917; Kolmogorov 1931). Moreover, the first-passage distribution function for the drift-free case was provided by Bachelier (1900) before Schrodinger (1915), and the effect of an absorbing barrier of Brownian motion was addressed by Bachelier (1900; 1901) prior to von Smoluchowski (1915; 1916). For a detailed summary of these early results see von Smoluchowski (1915; 1916).

Jevons (1878) pointed out that the chaotic movement of microscopic particles suspended in liquids had been noted long before Brown (1827) published his careful observations; however, it should be noted that Brown (1827) was the first to emphasize its ubiquity and to exclude its explanation as a biological phenomenon. A precise definition of the Brownian motion involves a measure of the path space that was first provided by Borel (1909) and constituted the basis of the formal theory of Wiener (1921a; 1921b; 1923).

Bachelier assumed stock price dynamics with a Brownian motion without drift (resulting in a normal distribution for the stock prices), and no time-value of money. The formula provided may be used to evaluate a European style call option. Later on, Kruczynski (1956) obtained the same results as Bachelier (1900). As pointed out by Merton (1973) and Smith (1976), this approach allows negative realizations for both stock and option prices. Moreover, the option price may exceed the price of its underlying asset.

Samuelson (1965) provided a rigorous review of the option valuation theory and pointed out that an option may have a different level of risk when compared with a stock, and therefore the discount rate used by Boness (1964) is incorrect. Samuelson and Merton (1969) provided a general equilibrium formula that depends on the utility function assumed for a typical investor. The Black and Scholes (1973) model is often regarded as either the end or the beginning of the option valuation history. Using two different approaches for the valuation of European style options, they present a general equilibrium solution that is a function of “observable” variables only, making therefore the model subject to direct empirical tests.

Literature Review

The concept of Brownian motion

The stock markets in the recent past have become an integral part of the global economy. Any fluctuation in this market influences our personal and corporate financial lives, and the economic health of a country. The stock market has always been one of the most popular investments due to its high returns (Kuo, Lee and Lee, 1996; Hassan and Nath, 2005). However, there is always some risk to investment in the Stock market due to its unpredictable behaviour. So, an ‘intelligent’ prediction model for stock market forecasting would be highly desirable and would be of wider interest. Reliable prediction of stock prices could offer enormous profit opportunities in reward and proactive risk management decisions. This quest has prompted researchers, in both industry and academia to find a way past the problems like volatility, seasonality and dependence on time, economies and rest of the market. The name Brownian motion derives from a very different route. In science it is given to the irregular movement of microscopic particles suspended in a liquid (in honour of the careful observations of the Scottish botanist Robert Brown, published in 1827). Einstein (1905) introduced his mathematical model of Brownian motion. Bachelier’s Brownian motion arises as a model of the fluctuations in stock prices. He argues that the small fluctuations in price seen over a short time interval should be independent of the current value of the price. Implicitly he also assumes them to be independent of past behaviour of the process and combined with the Central Limit Theorem he deduces that increments of the process are independent and normally distributed.

Brownian motion is often used to explain the movement of time series variables, and in corporate finance the movement of asset prices. Brownian motion dates back to the nineteenth century when it was discovered by biologist Robert Brown (1827) examining pollen particles floating in water under the microscope (Ermogenous, 2005). Brown observed that the pollen particles exhibited a jittery motion, and concluded that the particles were ‘alive’. This hypothesis was later confirmed by Albert Einstein in 1905 who observed that under the right conditions, the molecules of water moved at random. The first mathematical rigorous construction is due to Wiener in 1923 that is why Brownian motion is sometimes called as Wiener process (Ermogenous, 2006).

A common assumption for stock markets is that they follow Brownian motion, where asset prices are constantly changing often by random amounts (Ermogenous, 2005). The argument is built upon two key hypotheses: first, that the motion of the particles is caused by many frequent impacts of the incessantly moving molecules; second, that the effect of the complicated solvent motion can be described probabilistically in terms of very frequent statistically independent collision.

This concept has led to the development of a number of models based on radically different theories. Two common approaches to predicting stock prices are those based on the theory of technical analysis and those based on the theory of fundamental analysis (Fama, 1995). Technical theorists assume that history repeats itself, that is, past patterns of price behaviour tend to recur in the future. The fundamental analysis approach assumes that at any point in time an individual security has an intrinsic value that depends on the earning potential of the security, meaning some stocks are overpriced or under-priced (Fama, 1995). Many believe in an entirely different approach; the theory that stock market prices exhibit random walk. The random walk theory is the idea that stocks take a random and unpredictable path, making it near impossible to outperform the market without assuming additional risk. This theory casts serious doubts on the other methods of describing and predicting stock price behaviour. The GBM model incorporates this idea of random walks in stock prices (Reddy & Clinton, 2016).

Geometric Brownian motion has two components; a certain component and an uncertain component. The certain component represents the return that the stock will earn over a short period of time, also referred to as the drift of the stock. The uncertain component is a stochastic process including the stock's volatility and an element of random volatility (Sengupta, 2004). Brewer, Feng and Kwan (2012) describe the uncertain component to the GBM model as the product of the stock's volatility and a stochastic process called Weiner process, which incorporates random volatility and a time interval.

Empirical Review

With the fundamental discovery of Bachelier in 1900 that prices of risky assets (stock indices, exchange rates, share prices, etc.) can be well described by Brownian motion, a new area of applications of stochastic processes was born.

Islam and Nguyen (2020) present a comparative study for stock price prediction using three different methods, namely: Auto-regressive Integrated Moving Average (ARIMA), Artificial Neural Network and Stochastic process – GBM. Each of the methods is used to build predictive models using historical stock data collected from Yahoo Finance. Finally, output from each of the models is compared to the actual stock price. Empirical results show that the conventional statistical model and the stochastic model provide better approximation for next-day stock price prediction compared to the neural network model.

Rahul and Bidydhara (2020) stressed that Geometric Brownian Motion model is a mathematical model used for forecasting the future stock price and highly accurate as compared to other model and also gives high returns. They further stated that it helps the investors to take further decisions on their investment. Before forecasting the stock price using Geometric Brownian Motion model, Kolmogorov-Smirnov test and Q – Q plot technique were conducted on the sample data to conclude that the data are normally distributed and feasible to forecast. The algorithm starts from calculating the stock returns, drift and volatility to predict the return distribution at specific time 't'. Simulations were performed using log volatility equation with the closest behavior to the S&P BSE closed stock price. The closest forecast simulation with actual stock closed price is selected with the more precise value of drift and volatility to proceed in Geometric Brownian Motion model. In order to determine the forecast accuracy as well as performance of the model

Mean Absolute Percentage Error (MAPE) is calculated. Since $MAPE < 10\%$ i.e. 5.41 %, it implies that Geometric Brownian Motion model is highly accurate and an appropriate model for forecasting stock price.

Parungrojat and Kidsom (2019) explore, compare and evaluate the predictive power of the Geometric Brownian Motion (GBM) and the Monte Carlo Simulation technique in

forecasting the randomly selected 10 listed stocks in the SET50 of the Stock Exchange of Thailand (SET). The results shows that for the highest precision +/-0.5% of predicted 45 days returns, the percentage of accuracy is at the highest of around 5% (or 500 times in 10,000 trials) for both GBM and Monte Carlo Simulation. It can be concluded that model accuracy in predicting end period returns is limited. Especially, predictive powers of the models are declining towards the longer the evaluated timeframe. Comparing GBM and Monte Carlo Simulation in term of percentage of accuracy in predicting the end period returns, the two techniques are indifferent. For the predictive power of movements in prices, the GBM is a preferred technique. Besides, Monte Carlo Simulations yields a better accuracy especially in a longer period of evaluated timeframe. In conclusion, both techniques can predict stock prices within a highly accurate range. Thus, the techniques can be applied for stock price forecasting with limits mentioned.

Suganthi and Jayalalitha (2019) stated that financial instability estimates the changes of the cost of a monetary instrument. It is a proportion of properties of the Stock prices stability. Fractal investigations are used to assess the money related instability. Forecasting of stock prices acts as an important challenge based on the Random Walk theory. This paper deals with the comparison of two years of stock prices, 2013 -2014 and 2017 (June to Nov). Explain the instability by the method of Box-Counting technique to find the Fractal dimensions of the geometric Brownian motion based on the Random Walk defective value. This creates the possibility that Fractalijm measurement is related with the monetary unpredictability. It is an essential instrument for both money related investigators and Financial specialists.

Farida, Affianti, and Putri (2018) use geometric Brownian motion to predict the future price of stock. The phase that done before stock price prediction is determine stock expected price formulation and determine the confidence level of 95%. On stock price prediction using geometric Brownian motion model, the algorithm starts from calculating the value of return, followed by estimating value of volatility and drift, obtain the stock price forecast, calculating the forecast MAPE, calculating the stock expected price and calculating the confidence level of 95%. Based on the research, the output analysis shows that geometric Brownian motion model is the prediction technique with high rate of accuracy. It is proven with forecast MAPE value $\leq 20\%$.

Brodd and Djerf (2018) investigate how Monte Carlo simulations of random walks can be used to model the probability of future stock returns and how the simulations can be improved in order to provide better accuracy. The implemented method uses a mathematical model called Geometric Brownian Motion (GBM) in order to simulate stock prices. Ten Swedish large-cap stocks were used as a data set for the simulations, which in turn were conducted in time periods of 1 month, 3 months, 6 months, 9 months and 12 months. The two main parameters which determine the outcome of the simulations are the mean return of a stock and the standard deviation of historical returns. By varying the assumptions regarding price distribution with respect to the size of the current time period and using other weights, the method could possibly prove to be more accurate than what this study suggests. Monte Carlo simulations seem to have the potential to become a powerful tool that can expand our abilities to predict and model stock prices.

Nur Aimi Badriah, Siti Nazifah, and Maheeran (2018) forecast share prices for one month using geometric Brownian motion. The purpose of the study is to identify the best duration of historical data and forecast days in order to accurately forecast share prices of companies in Bursa Malaysia. This study focused on 40 listed companies in Bursa Malaysia from the top gainers list. It was found that 65 historical days could forecast the share prices for 21 days accurately.

Liden (2018) used a Geometric Brownian Motion (GBM) to predict the closing prices of the Apple stock price and also the S&P500 index. Additionally, closing prices have also been predicted by using mixed ARMA (p,q)+GARCH(r, s) time series models. Using 10 years of historical closing prices between 2008 and 2018, the predicted prices have also been compared to observed stock prices, in order to evaluate the validity of the prediction models. Predictions have been made using Monte Carlo methods in order to simulate price paths of a GBM with estimated drift and volatility, as well as by using fitted values based on an ARMA(p,q)+GARCH(r, s) time series model. The results of the predictions show an accuracy rate of slightly above 50% of predicting an up- or a down move in the price, by both using a GBM with estimated drift and volatility and also a mixed ARMA(p,q)+GARCH(r, s) model, which is also consistent with the results of K. Reddy and V. Clinton (2016).

Methodology

Research Design

Empirical research design is applied in this research since it is the most relevant form for time series data analysis, and the nature of the data analysis is determined by the actual behaviour of the financial data rather than a pre-conceived notion.

Population and Sample Design

The population of this study is made-up of the eight (8) sectors of The Nigerian stock Exchange. The selection of the sample size is judgmental. A sample of one stock is chosen from each of the sectors. To form a round figure of ten (10); a stock was taken from the financial sector, (as that sector has the highest number of listed (quoted) firms), while another stock was taken from the construction sector as the sector has the most frequently (continuously) traded stocks in the Stock Exchange. The ten (10) stocks include: Okomu palm oil – agricultural sector; J. Berger – construction; UPDC real estate investment trust – real estate; Dangote sugar – consumer goods; Guarantee trust bank – finance; Axamansard insurance (mansard) – finance; Neimeth – health care; Chams – ICT; Dangote cement – industrial; Seplat – oil & gas. The sample is taken from different sectors of The Nigerian Stock Exchange. There are eight (8) sectors in The Nigerian Stock Exchange. They are: Financials. The financial sector consists of banks, investment funds, insurance companies and real estate firms, among others. 2. Utilities. The utilities sector consists of electric, gas and water companies as well as integrated providers. 3. Consumer Goods. 4. Consumer Services. 5. Energy (Oil & Gas). 6. Healthcare 7. Industrials. 8. Technology.

Data Collection

Data is collected from ten (10) stocks quoted on The Nigerian Stock Exchange; daily stock prices data were obtained from the exchange database over the period four (4) years. The start date for the simulation was 1st January, 2015 and end 31st December, 2018, which was chosen to avoid any effects of seasonality in stock prices. The stock prices (open, high, low and close) of the companies are used to gather the data.

Model specification - Application of geometric Brownian motion (GBM)

The GBM method is used to forecast stock prices of the selected companies using the Monte Carlo simulation in the EXCEL software to determine the accuracy and effectiveness. For the GBM method, the procedures are as follows:

1. The data are tested for normality using the computer software.
2. The daily drift, daily volatility and the average drift are determined using the formula shown below:

$$\text{Daily Rate of Return} = \frac{\text{Annual Rate of Return}}{\text{No. of trading days in the year}} \quad (3.1)$$

$$\text{Daily Volatility} = \frac{\text{Annual Volatility}}{\sqrt{\text{No. of trading days in a year}}} \quad (3.2)$$

$$\text{Average Drift} = \text{Daily Rate of Return} - 0.5 \times \text{Daily Volatility}^2 \quad (3.3)$$

3. The value of the random number generated from probability distribution, ϵ , is determined using the EXCEL function of `NORM.S.INV(RAND())`. This function gives a random number from the normal distribution table.
4. Once all variables are known, the future stock value is determined using the Geometric Brownian motion formula as shown below:

$$S(t) = S(0)e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\epsilon(t)} \quad (3.4)$$

Where:

$S(t)$ = future stock value

$S(0)$ = initial stock value

μ = daily drift

σ = daily volatility

ϵ = value from probability distribution

Model Assumptions

Generally, the returns can be assumed as a random variable, which has closed enough to a normal distribution with a non-zero mean and standard deviation. So the distribution of asset returns can be defined as follows:

$$R_i = \frac{S_t - S_{t-1}}{S_{t-1}} = \text{Mean} + \text{Standard deviation} \times \varphi \quad (3.5)$$

Statistically, it is difficult to measure mean scale of the distribution with the smaller parameter δt which represents the time gap between assets. The drift rate or growth rate μ of the distribution can be assumed as a constant and defined as equation (3.6).

$$\text{Mean} = \mu \delta t \quad (3.6)$$

The volatility (standard deviation) of the distribution is significant and elusive quantity in the theory of derivatives. The standard deviation of the asset returns over a time step δt is given as equation (3.7), (Chang, Lima and Tabak, 2004).

$$\text{Standard deviation} = \sigma \delta t^{1/2} \quad (3.7)$$

Putting these scalings explicitly from equations (3.6) and (3.7) into asset return model represents in equation (3.5), then, we have:

$$R_i = \frac{S_t - S_{t-1}}{S_{t-1}} = \mu \delta t + \sigma \delta t^{1/2} \quad (3.8)$$

The term dW_t be a random variable, from normally distribution with mean zero and variance δt . So equation (3.8) can be simplified as follows:

$$dS_t = \mu S_t \delta t + \sigma S_t dW_t \quad (3.9)$$

Integrating equation (3.9) with respect to t ;

$$\int_0^t \frac{dS_t}{S_t} = \mu t + \sigma W_t \quad (3.10)$$

Where; let we assume that, $w_0 = 0$. It is clear that, term S_t under the Ito process. So we used Ito's calculation for our further study:

$$d(\ln S_t) = \frac{dS_t}{S_t} - \frac{1}{2} \mu^2 dt \quad (3.11)$$

Now, substitute equation (3.9) into the equation (3.11) we get;

$$d(\ln S_t) = \mu dt + \sigma dw_i - \frac{1}{2} \sigma^2 dt \tag{3.12}$$

Integrating both sides with respect to t , we get:

$$S_t = S_0 \exp\left[\left(\mu - \frac{1}{2} \sigma^2\right) t + \sigma w_i\right] \tag{3.13}$$

Where, $w_t = X_t - X_0$. Equation (3.12) indicated continuous stochastic process of Geometric motion that we used for simulation for the forecast of stock market indices.

Model Accuracy Testing

Time series forecasting considered as a technique which can be used for predicting future aspects of many operations. Numerous methods have been carried out by many research works to accomplish their goals. In this study, Mean Absolute Percentage Error (MAPE) is used to compare the prediction accuracy of the model. The accuracy model as defined by Wang, Wang and Zhang (2012) is as follows:

$$\epsilon_{MAPE} = \frac{1}{M} \sum_{j=1}^M \frac{|X_A - X_P|}{X_A} \tag{3.14}$$

Where X_A and X_P represent the actual value and predicted value of the indices respectively. The table below represents the scale of judgment of forecast accuracy regarding error (MAPE) and clearly indicated that minimum values of MAPE make more accuracy for forecasting future prediction (Omar and Jaffar, 2011).

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Dang-Cement	0.141729	4.413393	0.0000
Seplat	0.062585	1.673038	0.0037

Source: Author

The normality assumption does not hold for five of the companies (Seplat, Dang-Cement, Neimeth, Chams and J-Berger). However, for the unadjusted K-S statistic (with smaller values), the normality preposition cannot be rejected in any of the cases.

Based on Kolmogorov – Smirnov normality test it can be concluded that the stock price is normally distributed and feasible to forecast stock price using the data. The unit period of time, which we used in this simulation is of one-day length (1 day = 1/252 ≈ 0.004 year). On the average there are 252 trading days in the year. The value of the stock volatility and its drift can be respectively estimated according to the general GBM model. The results are shown in table 4.2.

The first thing we did was to estimate the expected level of return, and the volatility of the stock. The expected level of return can be estimated by calculating the mean of the historical returns. The volatility can be estimated by calculating the standard deviation of the historical returns. From the expected level of return and from the volatility of the stock we set up a probability distribution which attempts to model the behaviour of the stock. The simulation itself is the act of generating random numbers out of the probability distribution function. By sampling, say ten values from the distribution, we get a sense of how the stock could potentially behave within the next ten days.

Table 4.4 Proposition 1 Summary Results

	Correlation		Mean Absolute Percentage Error			
	12-Month	24-Month	36-Month	12-Month	24-Month	36-Month
Mean	-0.0149	0.000921	0.170405	0.502666	0.462861	0.590441
mean-absolute	0.014902	0.000921	0.170405	0.502666	0.462861	0.590441
Median	-0.01481	5.89E-05	0.366873	0.603449	0.393031	0.514993
median-absolute	0.014812	5.89E-05	0.366873	0.603449	0.393031	0.514993
Range	0.238638	1.788266	1.568857	0.913048	0.80997	0.93167
Min	-0.14158	-0.91565	-0.73448	0.008695	0.047987	0.14347
Max	0.097057	0.872616	0.834376	0.921742	0.857958	1.075141
standard-deviation	0.075004	0.734138	0.536576	0.328501	0.267266	0.336565

Note that the MAPE value between 1% and 10% indicates accurate forecast, 11% and 20% shows good forecast, 21% and 50% deemed reasonable and above 50% is inaccurate forecast. (see, Abidin & Jaffar, 2014)

Source: Author

Accuracy and Predictive Power of the Model

(i) Accuracy of the Model

The model accuracy depends on the percentage of the error. The smaller the MAPE value, the more accurate the forecasting model is. A scale of judgment of forecasting accuracy with MAPE is that MAPE value between 0% and 10% indicates accurate forecast, 11% and 20% shows good forecast, 21% and 50% deemed reasonable and above 50% is inaccurate forecast. (see, Abidin & Jaffar, 2014).

To give more confidence to the researchers, we measured the accuracy of the forecast model by looking at the MAPE value. Table 4.3 shows the forecast price and actual price from 2015 to 2018. The value of the MAPE is 50% and below for the one to two year holding periods, and above 50% for the three year holding period. Meaning that the GBM model is a reasonable predictive model for one or two years, but for three years, it is an inaccurate predictor. It was found that MAPE was lowest over simulation periods of one-holding period and two-holding period, but the error tended to increase when longer horizons were considered. Due to random behaviour of stock price, Geometric Brownian motion model is highly suitable for short term forecasting.

It is also observed in table 4.4 that the absolute median and standard deviation are smaller for the short holding period than for the long holding period. This suggests that the correlation between the predicted and the actual prices is less volatile in the short holding period than in the long holding period. Alternatively, the relationship between the stimulated value and real value of these stocks is more stable in the one-year holding period than in the two- to three-year holding periods.

The mean correlation over the short-term is slightly negative and grows positive as the simulation period is increased. This means that for one-year holding and two-year holding, predictions simulated prices move in the opposite direction with the real prices. For periods of three years or longer they correlate and begin to follow the prices more accurately. This could be a result of the certain component of the GBM model compensating for negative random fluctuations, as stock prices tend to increase over time. Looking at the median correlation leads to similar results as this is also negative for short periods and becomes positive after one year. The absolute mean and standard deviation are lowest for one year predictions suggesting that there is less variability over this prediction horizon.

(ii) Predictive power of the model

The predictive power of the model is declining towards the longer the evaluated time frame proven by the higher value of the mean absolute percentage error.

Portfolios formation

We form quartile portfolios of 30%, 40% and 30%, based on high, moderate and low respectively and the formation period is 2015. The portfolios are classified by volatility, expected return and market capitalization. The holding periods are 12 months (which is 2016), 24 months (2016-2017) and 36 months (2016-2018).

Testing the hypothesis that there is no significant difference between actual and predicted prices for portfolios classified by volatility, a quartile portfolio of 30%, 40% and 30% (highly volatile, moderate and low portfolios) respectively was formed based on the value of the stock standard deviations, and the formation period is 2015. The holding periods are 12 months (which is 2016), 24 months (2016-2017) and 36 months (2016-2018). The results of the comparison in each of these holding and three portfolios are shown in table 4.5

Table 4.5 Summary Results for the Test of Proposition 2 based on Correlation

Portfolio classification	12 months	24months	36 months
High Volatile Portfolio	0.632449	0.996749	-0.33139
		1931	

Moderate Volatile Portfolio	0.999967	0.999988	0.999995
Low Volatile Portfolio	0.999828	0.999935	0.999941

Source-Author

The correlation coefficient between the predicted and real prices for high volatile portfolio is positive for 12 and 24-month holding period, but negative for the 36-month holding period. This suggests that for a highly volatile portfolio, predicted and real prices move in the same direction; there is convergence and history can repeat itself. No randomness (differences) since the prices of these stocks can be predicted based on past information. However, for three year holding period, the real prices of stocks for volatile portfolio are significantly different from the predicted. Meaning that stock of volatile portfolio are stochastic when held for three years but deterministic when held for one or two years. Correlation coefficients with respect to moderate and low volatile portfolios are very strong for all the three holding periods. This suggests inefficiency and repeatability of past prices in the future.

We also form portfolios based on the expected returns of stocks, and conduct the test to show if there is difference between real and stimulated stock prices. The summary results are presented in table 4.6.

Table 4.6 Summary Results for the Test of Preposition 3 based on Correlation

	12 months	24months	36 months
High Expected Return Portfolio	0.996348	0.996092254	0.996651517
Moderate Expected Return Portfolio	0.994899	0.998989672	0.99942051
Low Expected Return Portfolio	0.969479	0.981217239	0.637766001

Source-Author

The table (4.6) shows that correlation coefficients for three portfolios and in each holding period are positive, and approximately 1. Thus, we have evidence that there is approximately perfect correlation between the predicted prices and the stimulated. Meaning that, real prices of portfolios chosen by expected returns are truly represented by predicted prices. The technical assumption is fair for stocks of portfolios using expected return.

Furthermore, a hypothesis that there is no significant difference between actual and predicted prices for portfolios classified by Market Capital was tested. We present the result of the test on preposition four in table 4.7 below.

Table 4.7 Summary Results for the Test of Preposition 4 based on Correlation

	12 months	24months	36 months
High Capitalized Portfolio	0.973531	0.989879	0.994015
Moderate Capitalized Portfolio	0.998766	0.996531	0.996467

Low Capitalized Portfolio	0.999928	0.999857	0.999847
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Source-Author

As shown in table 4.7, the correlation coefficient is close to 1 for the holding period in each portfolio. This classification does not support the randomness of stock prices since historical/previous prices are perfectly positively correlated with forecast prices. In essence, on the basis of portfolio formation using market capitalization, predicted prices converge to real prices.

Discussion of Findings

Generally, the test attempts to investigate whether or not there is a significant difference between actual and simulated prices using the GBM model by looking at the stocks on an individual basis. The expected returns of GBM model are independent of the value of the process, which agrees with what we would expect in reality; and shows the same kind of movement in its paths as we see in real stock prices.

Comparing the predicted with the actual, cases of significant differences were observed. From Table 4.4, it is realized that when the average returns predicted by the model are compared to the actual average returns, there are significant differences between the predicted and actual in traded stocks.

Compared with the previous works, the results confirm the work of Abidin and Jaffar (2012) which concluded that the GBM suits for short-term investment; and also echoes the work of Kawinpas et al (2015), concluding that the shorter the forecasting period the better the results.

Most of the prior studies have suggested that there is a weak relationship between the two variables. We have reported a negative correlation during short periods of simulation, which becomes positive with longer forecast horizons. Noise or volatility in the market makes simulated stock price and actual stock price to have a negative correlation in the short term, whereas stock prices stabilize to its mean value in the longer run causing a positive correlation between simulated and actual prices.

Conclusion

Generally speaking, modeling of stock prices is about modeling new information about stocks. In this study modeling has been realized through the quantification of a stochastic part in the general expression of the model. The results of the simulation performed in this study do not always match those of the theoretical model even if the assumptions on which the model is based meet the financial market rules.

The analysis of simulation results provides important thing: the correlation between the predicted prices and the actual prices is less volatile in the short holding period than in the long holding period. Alternatively, the relationship between the stimulated value and real value of these stocks is more stable in the one-year holding period than in the two to three year holding periods.

Recommendation

It is recommended that the Geometric Brownian Motion model should be used to forecast daily stock prices [over short period as it gives more accurate result].

The Technical Analyst whose primary motive is to make gain at the expense of other participants should identify high volatile portfolio in any holding period for effective prediction. Regulatory authorities in the market should focus on long-term stability by holding reserves that can cushion off the effects of illiquidity, particularly in the time of extreme turbulence. Investors with long-range holding position as investment strategy should concentrate more on low capitalized stocks rather than stocks with large market capitalization. However, It is also suggested that they should diversify to more efficient portfolio with combination of low and large capitalized stocks so the loss or the gain resulting from market inefficiency can be reduced optimally.

Contribution to knowledge

As the random character of the GBM model stands out in the short term, this approach presents the limitation of not allowing the investigation of any behavior trend. In the long run, however, this is not observed, and a trend can be studied. This study illustrates both scenarios by presenting the most random character in the short term and a trend in the long term, for the same simulation. The study reveals that the GBM model can diverge if the time series is built for long-term period, driving the price () to infinity; in that sense some simulations that follow the model may not be realistic.

One would intuitively assume that stocks with higher expected returns would perform better than those with lower expected returns. However, as shown in table 4.6, this may not always be the case. Forecasts for the portfolio show that over the simulation period, correlation coefficients for the three formed portfolios are positive and approximately one (1) despite having high-to-moderate expected return. Meaning that, real prices of portfolios chosen by expected returns are truly represented by predicted prices, therefore, the technical assumption is fair for stocks of portfolios using expected returns.

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